Subject: Leaving Certificate Maths

Teacher: Mr Murphy

Lesson 18: Calculus III

18.1 Learning Intentions

After this week's lesson you will be able to;

- Calculate the equation of a circle
- Extract the centre and radius from a circle x°+y'+2gx+2fy+c=0
- Establish the location of a point relative to the circle
- Describe other types of centres Find the P.O.I. of a circle and a line
- Calculate the equation of a tangent to a circle
- Prove that circles are touching

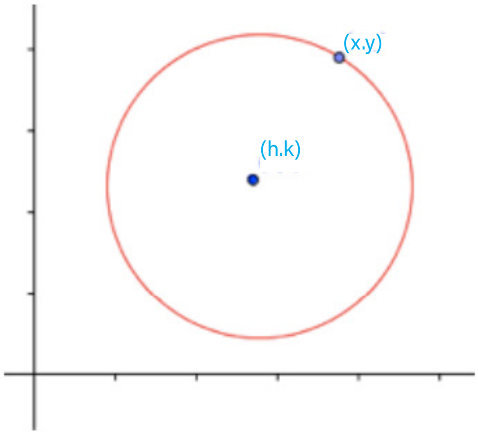
18.2 Specification

2.2 Co-ordinate geometry	– use slopes to show that two lines are • parallel • perpendicular – recognise the fact that the relationship $ax + by + c = 0$ is linear – solve problems involving slopes of lines – calculate the area of a triangle – recognise that $(x-h)^2 + (y-k)^2 = r^2$ represents the relationship between the x and y co-ordinates of points on a circle with centre (h, k) and radius r – solve problems involving a line and a circle with centre (0, 0)	– solve problems involving • the perpendicular distance from a point to a line • the angle between two lines – divide a line segment internally in a given ratio m: n – recognise that $x^2+y^2+2gx+2fy+c=0$ represents the relationship between the x and y co-ordinates of points on a circle with centre $(-g,-f)$ and radius r where $r = \sqrt{(g^2+f^2-c)}$ – solve problems involving a line and a circle

18.3 Chief Examiner's Report

A	4	18·6	75	2	Co-ordinate geometry - circle

18.4 Equation of teh Circle



using the distance formula and the given points, establish the distance between the two points.

$$r^2 = (x-h)^2 + (y-k)^2$$

Again, what you have done is establish the formula for calculaating the equation of a circle with a non-zero cnetre and radius r. So, if we take a picture of a circle see if you can extract the important information from it a establish the equation of this circle.

18.5 Alternative Version of the Equation

Sometimes the equation can be given in the following form:

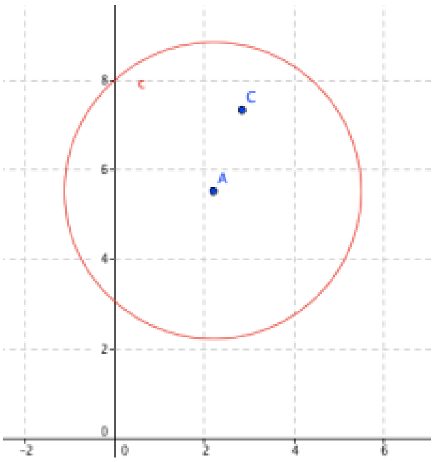
$$x^2 + y^2 + 2gx + 2fy + c = 0$$

Where the centre is (-g, -f) and the radius:

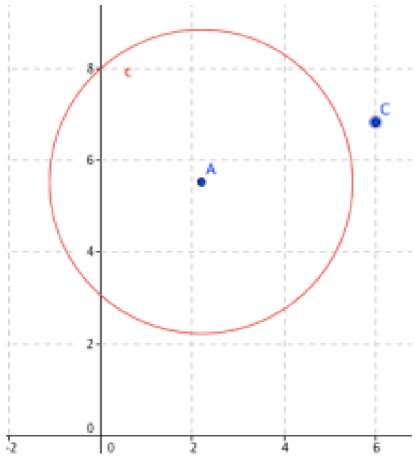
$$r = \sqrt{g^2 + f^2 - c}$$

So, in extracting the centre from this form of equation we half the coefficients of both the x and y and change their signs.

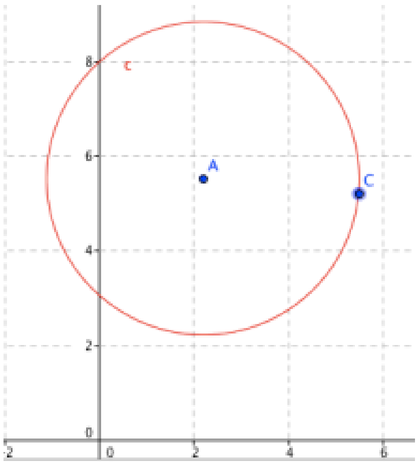
18.6 Points Location Relative to a Circle



$$r < |AC|$$

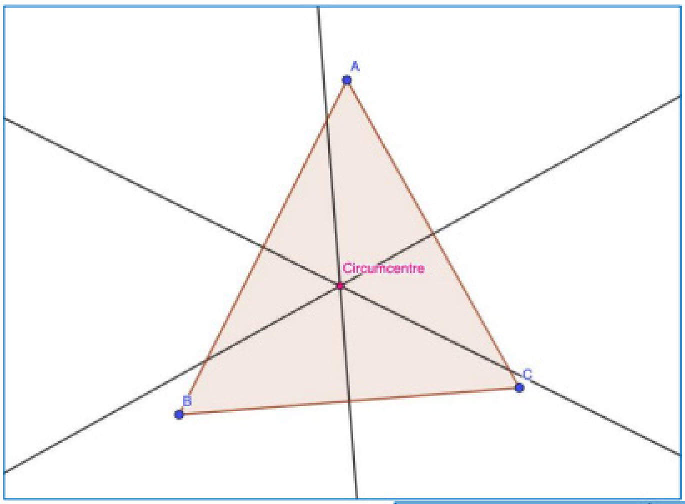


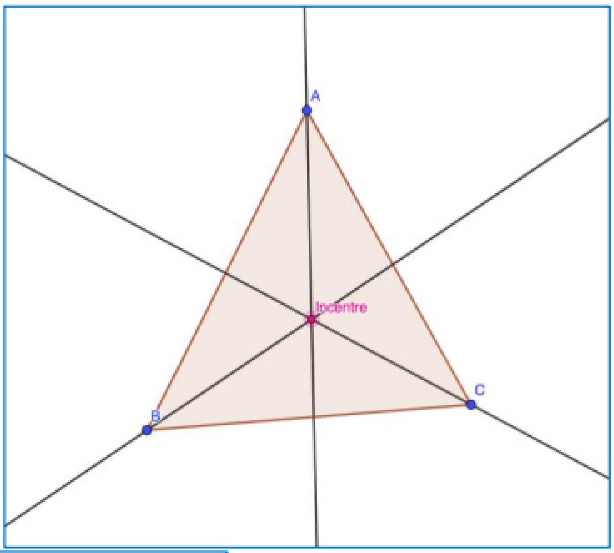
$$r > |AC|$$

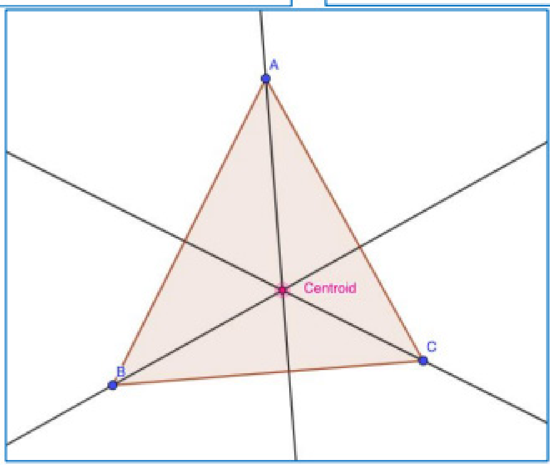


$$r = |AC|$$

18.7 Other Centres in Geometry

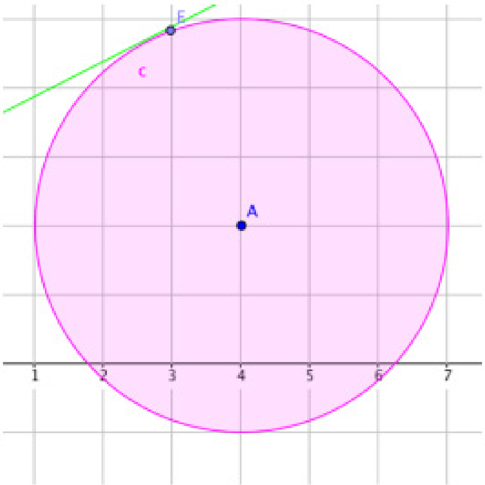




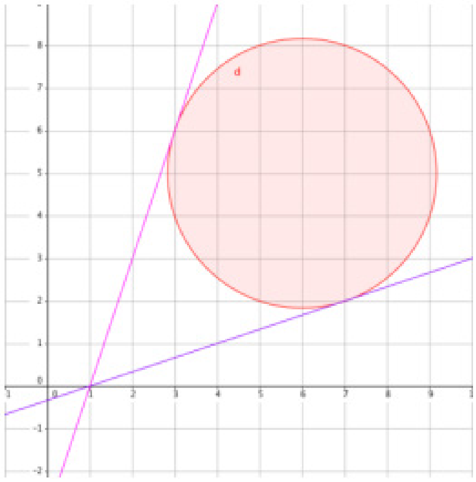


18.8 Equation of a Tangent

With a tangent and a circle, we can have two situations.



Point on the circle

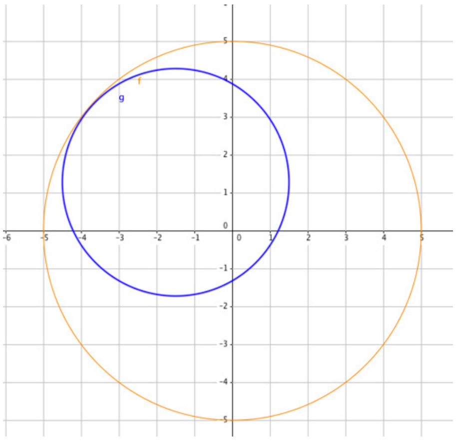


Point outside the circle

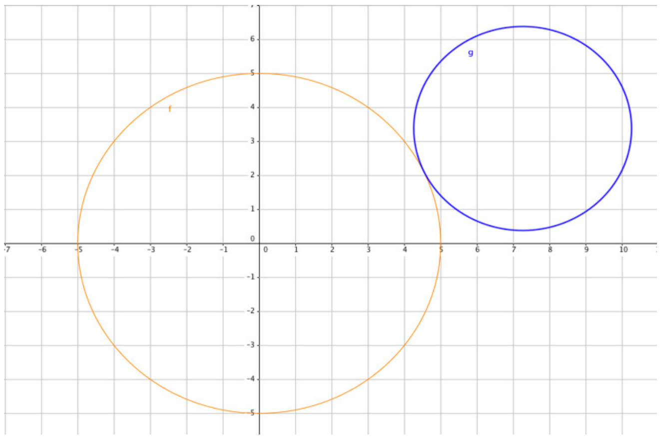
Points on the Cicle:

Points outside the circle $x^2 + y^2 - 12x - 10y + 51 = 0$:

18.9 Circle in Contact



Internal



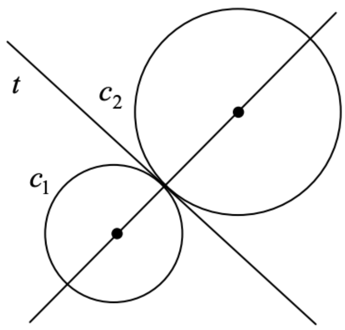
External

Internal: Distance between the two ccentres is equal to the difference in teh two radii.

External: Distance between the two centres is equal to the sum of the two radii.

Question:

The circles c_1 and c_2 touch externally as shown.



(a) Complete the following table:

Circle	Centre	Radius	Equation
c_1	$(-3, -2)$	2	
c_2			$x^2 + y^2 - 2x - 2y - 7 = 0$

(b) (i) Find the co-ordinates of the point of contact of c_1 and c_2.

18.10 Recap of the Learning Intentions

After this week's lesson you will be able to;

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18.11 Homework Task

Two circles s and c touch internally at B, as shown.

(a) The equation of the circle s is

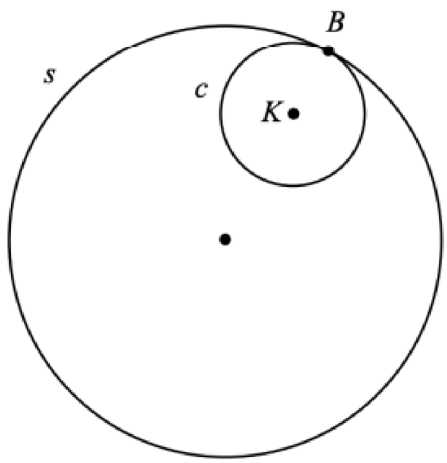
$(x - 1)^2 + (y + 6)^2 = 360.$

Write down the co-ordinates of the centre of s.

Centre: ____________________

Write down the radius of s in the form $a\sqrt{10}$, where $a \in \mathbb{N}$.

Radius: ____________________



(b) (i) The point K is the centre of circle c.

The radius of c is one-third the radius of s.

The co-ordinates of B are (7, 12).

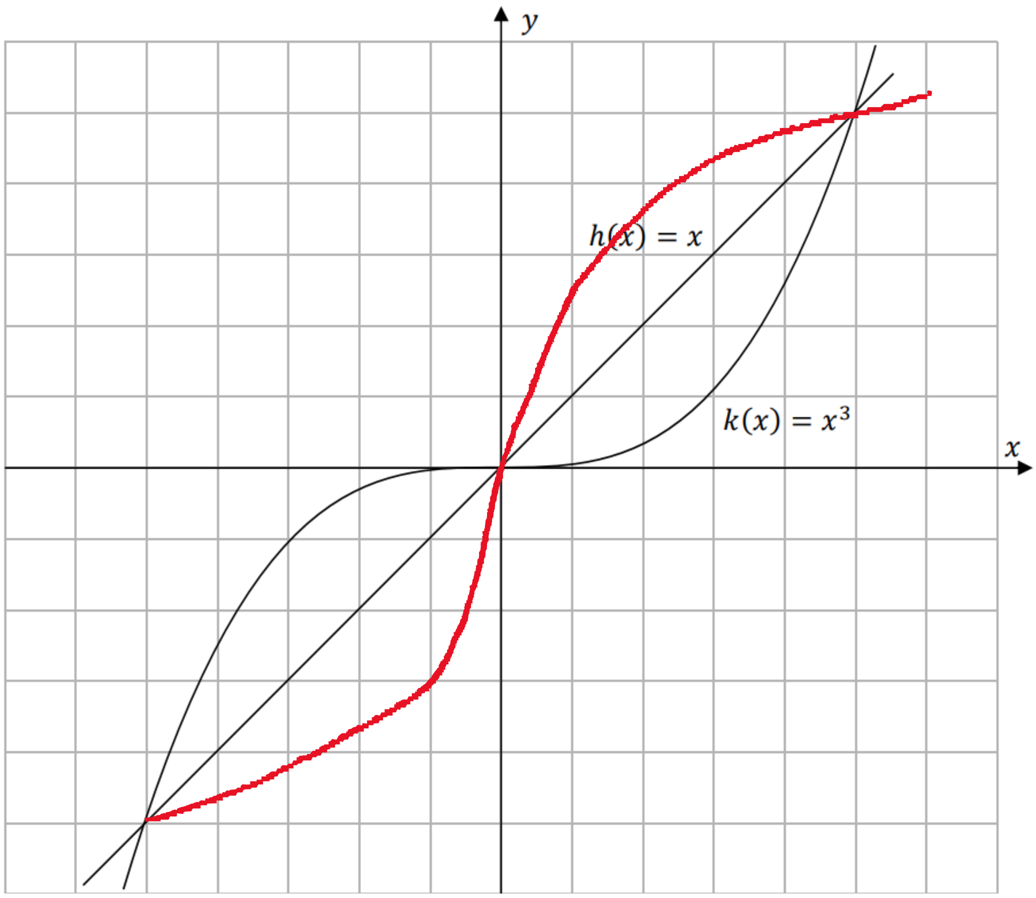
Find the co-ordinates of K.

(ii) Find teh equation of c.

(c) Find the equation of the common tangent at B.
Give your answer in the form $ax + by + c = 0$, where $a, b, c \in \mathbb{Z}$.

18.12 Solutions to 17.10

Parts of the graphs of the functions $h(x) = x$ and $k(x) = x^3$, $x \in \mathbb{R}$, are shown in the diagram below.



(a) Find the co-ordinates of the points of intersection of the graphs of the two funcations.

$$x^3 = x$$

$$x^3 - x = 0$$

$$x(x^2 - 1) = 0$$

$$x(x-1)(x+1) = 0$$

$$x = 0 \text{ or } x = -1 \text{ or } x = 1$$

Sub these into either y= to get the y coordinate

P.O.I. = $(-1,-1), (0,0), (1,1)$

(b) i. Find the total ara enclosed between the graphs of the two functions.

$$2\int_0^1 x - x^3 dx$$

$$= 2\left[\frac{x^2}{2} - \frac{x^4}{4}\right]_0^1$$

$$= 2\left(\left(\frac{1^2}{2} - \frac{1^4}{4}\right) - \left(\frac{0^2}{2} - \frac{0^4}{4}\right)\right)$$

$$\frac{1}{2}\ sq.units$$

ii. On the diagram on teh previous page, using symmetry or otherwise, draw the graph of K^{-1}, the inverse function of K.